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Effect of Gas-Properties Evaluation Method on the Optimum Point of Gas Turbine Cycles

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Abstract

Recent work has revealed that the assumption regarding the behavior of gases (perfect, ideal, real) and, consequently, the way their properties are evaluated may alter critically the picture obtained about the performance of gas turbine systems. This fact prompted an investigation of how the aforementioned assumption may affect the optimal design point of gas turbine systems. The present study is restricted to a comparison between the ideal and perfect gas assumption. Three systems have been selected for study and three optimization problems have been formulated and solved for each system: two thermodynamic and one thermoeconomic. The results demonstrate that the method used for the evaluation of properties of gases has a very significant effect on the optimal point of each system.

Keywords: Gas properties, gas turbine cycles, optimization, thermoeconomics.

1. Introduction

It is common knowledge that the efficiency of a simple gas turbine cycle increases monotonically with the maximum cycle temperature for the constant pressure ratio (Haywood 1987). In order to be more specific, the efficiency of the air standard cycle (assumption of perfect gas with no change of mass flow rate due to fuel addition and no pressure losses in the ducts and the combustion chamber) is given by the equation:

$$\eta_A = \frac{\eta_C \eta_T \eta_J \tau_3 - \left(r^k - 1\right)}{\eta_C \left(\tau_3 - 1\right) - \left(r^k - 1\right)} \qquad (1)$$

where

$$\eta_J = 1 - \frac{1}{r^k} \qquad (2)$$

is the Joule cycle efficiency, i.e. the ideal cycle with isentropic compression and expansion and no losses. Starting with equation. (1) it is easily proved that, if the turbine temperature is increased, keeping the pressure ratio constant, the thermal efficiency of the air standard cycle increases continuously and asymptotically it reaches the limit:

$$\lim_{\tau_3 \to \infty} \eta_A = \eta_T \eta_J \qquad (3)$$

It was tacitly assumed that the general trend was the same even if a change of specific heat capacity or of the mass flow rate due to fuel addition was considered. Thus, it was a surprise to read in Horlock (2003) that, if the assumption of a working substance of constant quality and quantity is relaxed, then the behavior changes drastically; for a constant pressure ratio, the efficiency initially increases with the turbine-inlet temperature, it reaches a maximum value and then it decreases. Detailed studies of these effects appear in Horlock (2000 and 2001) and Guha (2003). This remark prompted the investigation reported here.

Many publications on optimization of gas turbine cycles, e.g. Frangopoulos (1988, 1992 and 1994), Valero et al. (1994), are based on the assumption of perfect gas with different values for the specific heats of air and exhaust gases, in order to decrease the inaccuracy. After the aforementioned, the question arises: "How is the optimum point affected if the properties of gases are evaluated with a higher accuracy?" An answer to this question is attempted in the following, using as examples three different system configurations.

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2. Evaluation of Gas Properties

A clarification of terminology is useful at this point. The specific heat capacities of a real gas are functions of both temperature and pressure:

$$c_p = c_p(p, T), \qquad c_v = c_v(p, T) \tag{4}$$

For an *ideal gas*, they are functions of the temperature only:

$$c_p = c_p(T), \qquad c_v = c_v(T) \tag{5}$$

For a *perfect gas*, they are constant:

$$c_p = \text{const.}, \qquad c_v = \text{const.} \tag{6}$$

This 'textbook material' is repeated here because it is often written in related publications that 'real-gas' effects are studied, while in fact the gases are considered ideal. Thus, the reader should be careful.

In the present work, for the perfect gas model the values

$$c_{pa} = 1.004 \text{ kJ/kgK}, \quad c_{pg} = 1.170 \text{ kJ/kgK}$$

have been considered. For the ideal gas model, it is considered that air consisting of N_2, O_2, CO_2 and H_2O is compressed and then reacts with a fuel having the general composition $C_\alpha H_\beta$ in a complete combustion to produce exhaust gases consisting of N_2, O_2, CO_2 and H_2O. For simplicity, minor constituents (such as CO, NO_x, etc.) due to additional reactions, dissociation, impurities or other reasons are not considered here. The properties of each species are evaluated by the following equations obtained from Gordon et al. (1994) and McBride et al. (2002):

$$\frac{\tilde{c}_{p0}}{\tilde{R}} = a_1 \cdot T^{-2} + a_2 \cdot T^{-1} + a_3 + a_4 \cdot T + a_5 \cdot T^2 + a_6 \cdot T^3 + a_7 \cdot T^4 \tag{7}$$

$$\frac{\tilde{h}_0}{\tilde{R}} = -a_1 \cdot T^{-1} + a_2 \cdot \ln(T) + a_3 \cdot T + \frac{1}{2} a_4 \cdot T^2 + \frac{1}{3} a_5 \cdot T^3 + \frac{1}{4} a_6 \cdot T^4 + \frac{1}{5} a_7 \cdot T^5 + b_1 \tag{8}$$

$$\frac{\tilde{s}_0}{\tilde{R}} = -\frac{1}{2} \cdot a_1 \cdot T^{-2} - a_2 \cdot T^{-1} + a_3 \cdot \ln(T) + a_4 \cdot T + \frac{1}{2} \cdot a_5 \cdot T^2 + \frac{1}{3} \cdot a_6 \cdot T^3 + \frac{1}{4} \cdot a_7 \cdot T^4 + b_2 - \ln(P) \tag{9}$$

The numerical values of the parameters a_i depend on the species and the temperature range, and are given in McBride et al. (2002).

The properties of air and exhaust gases are evaluated with the assumption that they are ideal mixtures; for example, the molar heat capacity is calculated by the equation:

$$\tilde{c}_p = \sum_i x_i \tilde{c}_{pi} \tag{10}$$

Thus, the perfect gas assumption of previous works has been replaced here with the ideal gas assumption. The effect of pressure is still considered negligible for the pressure ranges used in the systems that are studied here, as justified by values obtained for the compressibility factor. Furthermore, models for thermodynamic properties of real gases are not presently available for all the constituents and all the temperature and pressure ranges appearing in gas turbine cycles. Therefore, the real gas effect is left for future investigation.

3. Systems Studied

3.1 Description of the systems

Three systems have been selected in order to study the effect of the method used for property evaluation on the optimal design point.

System I consists of a simple, open-cycle gas turbine *(Figure 1)*. An approach for its thermodynamic and thermoeconomic optimization based on the perfect gas assumption has been presented in Frangopoulos (1988 and 1992).

System II is a cogeneration plant consisting of a regenerative gas turbine with an exhaust gas boiler producing saturated steam *(Figure 2)* of a given quality and quantity; it is the system of the CGAM problem (Valero et al. 1994 and Frangopoulos 1994).

System III is an inter-cooled, regenerative gas turbine with a twin spool gas generator and a power turbine *(Figure 3)*.

3.2 Mathematical Models of the Systems

3.2.1 Thermodynamic model of System I

The air temperature at the exit of the compressor and the exhaust gas temperature at the exit of the turbine are evaluated by the equations:

$$T_2 = T_1 \cdot \left[1 + \left(r_C^{\frac{k_{12}-1}{k_{12}}} - 1\right) \cdot \frac{1}{\eta_C}\right] \tag{11}$$

$$T_4 = T_3 \cdot \left[1 - \left(1 - r_T^{\frac{1-k_{34}}{k_{34}}}\right) \cdot \eta_T\right] \tag{12}$$

where

$$k_{ij} = \frac{\tilde{c}_{pij}}{\tilde{c}_{pij} - \tilde{R}} \tag{13}$$

$$\tilde{c}_{pij} = \frac{1}{T_j - T_i} \int_{T_i}^{T_j} \tilde{c}_p(T)\, dT \qquad (14)$$

Since k_{12} and k_{34} depend on the temperatures T_2 and T_4, respectively, equations. (11) and (12) are used in an iterative procedure in order to obtain the temperatures T_2 and T_4.

An energy balance in the combustion chamber gives the equation:

$$f H_u \eta_B = (1+f)(h_3 - h_0) - (h_2 - h_0) \qquad (15)$$

which can be solved for f, if the temperature T_3 is given. The composition of the exhaust gases is determined by the reaction of combustion for any specified fuel. Since the temperature T_3 and the composition of exhaust gases are interrelated through the temperature-dependent properties of the constituents, an iterative procedure is also applied here.

The system efficiency is given by the equation:

$$\eta_I = \frac{\dot{W}}{\dot{m}_f H_u} = \frac{(1+f)(h_3 - h_4) - (h_2 - h_1)}{f H_u} \qquad (16)$$

The specific work is also of interest:

$$w = \frac{\dot{W}}{\dot{m}_a} \qquad (17)$$

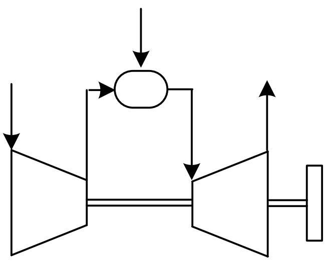

Figure 1. System I

3.2.2 Thermodynamic model of system II

Equations (11)-(15) and (17) are valid for System II, with a proper adjustment of certain numerical indexes. The effectiveness of the air pre-heater is given by the equation:

$$\varepsilon_X = \frac{h_3 - h_2}{h_{a5} - h_2} \qquad (18)$$

The subscript 'a5' is used in equation. (18) in order to make it clear that h_{a5} is the enthalpy of air at temperature T_5. There is no ambiguity about h_2 and h_3. The following efficiencies are defined for this system.

Net shaft-power efficiency:

$$\eta_{II} = \frac{\dot{W}}{\dot{m}_f H_u} = \frac{(1+f)(h_4 - h_5) - (h_2 - h_1)}{f H_u} \qquad (19)$$

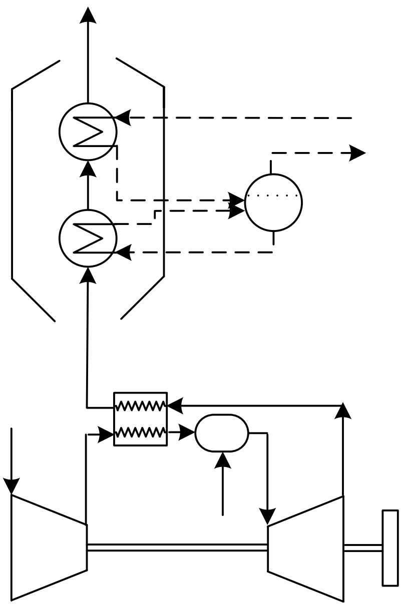

Figure 2. System II

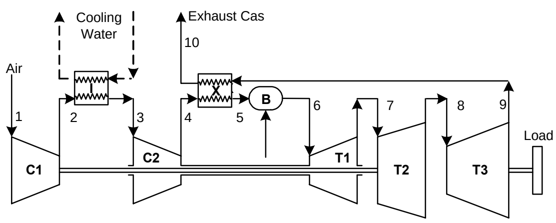

Figure 3. System III

Efficiency of providing the useful heat:

$$\eta_{II,Q} = \frac{\dot{Q}}{\dot{m}_f H_u} = \frac{\dot{m}_s (h_9 - h_8)}{\dot{m}_f H_u} \qquad (20)$$

Total efficiency:

$$\eta_{II,tot} = \eta_{II} + \eta_{II,Q} = \frac{\dot{W} + \dot{Q}}{\dot{m}_f H_u} \qquad (21)$$

The model of System II consists of many more equations, which are given in Ref. [9], but they are not repeated here due to space limitations.

3.2.3 Thermodynamic model of system III

For the compression, combustion and expansion processes, equations similar to those of System I are used. The effectiveness of the air pre-heater is given by an equation similar to equation. (18), with proper adjustment of the numerical indexes. In addition, the following equalities are taken into consideration:

$$\dot{W}_{C1} = \dot{W}_{T2}, \qquad \dot{W}_{C2} = \dot{W}_{T1} \qquad (22)$$

The division of the pressure ratio between the low-pressure and high-pressure spool is determined by an iterative procedure so that equation. (22) is satisfied.

The system efficiency is given by the equation:

$$\eta_{III} = \frac{\dot{W}}{\dot{m}_f H_u} = \frac{(1+f)(h_8 - h_9)}{f H_u} \qquad (23)$$

The specific work is given by equation. (17).

3.2.4 Thermoeconomic models of the systems

The cost functions for Systems I and II appear in Frangopoulos (1988, 1992 and 1994) and Valero et al. (1994). For System III, equations available for Systems I and II have been properly modified and used. Space limitations do not allow giving the complete set of equations here.

4. Performance of Systems with Alternative Methods for Evaluation of Properties

For the performance evaluation and for the optimization of the systems, certain values have been considered for the various parameters involved, which are given in TABLE I.

The first step in this investigation has been the study of the effect of properties evaluation on the simple cycle efficiency (System I). The results depicted in *Figure 4* are revealing: the perfect gas assumption, as expected, gives an efficiency continuously increasing with the turbine inlet temperature. With the ideal gas assumption and properties evaluated by equations. (7)-(10), the efficiency exhibits a maximum at a temperature of about 1600 K (for the parameter values considered here). Thus, the related results of Guha (2003) are reproduced to a very close approximation (small differences are due to different values of parameters and to the different sources of equations for evaluation of gas properties).

With the perfect gas model, the specific work increases continuously with turbine inlet temperature for a certain pressure ratio. This trend remains the same with the ideal gas model too.

In *Figures 5 and 6*, the effect of the gas model assumption on the system efficiency and specific work as functions of pressure ratio is shown. It is clarified that the graphs of *Figures 5B and 6B* correspond to the system of *Figure 2* but without the exhaust gas boiler.

The coordinates of the optimum points in *Figures 5 and 6* are given in TABLES II and III, respectively.

TABLE I. VALUES OF PARAMETERS.

System I	System II	System III
$r_B = 0.975$	$r_B = 0.975$	$r_{C1} = r_{C2}$
$\eta_B = 0.99$	$\eta_B = 0.99$	$r_I = 0.98$
$\eta_m = 0.99$	$r_{Xa} = 0.975$	$T_3 = T_1$
	$r_{Xg} = 0.965$	$r_B = 0.975$
Air	$\Delta T_{X\,min} = 20\,K$	$\eta_B = 0.99$
N_2: 77.82%	$\eta_m = 0.99$	$r_{Xa} = 0.975$
O_2: 20.68%	$r_R = 0.95$	$r_{Xg} = 0.965$
CO_2: 0.03%	$\Delta T_{p\,min} = 15\,K$	$\Delta T_{X\,min} = 20\,K$
H_2O: 1.47%	$T_{7\,min} = 373.15\,K$	$\eta_m = 0.99$
$T_{amb}=T_0=25C^o$	$\dot{m}_{st} = 14\ kg/s$	$r_R = 0.95$
$P_{amb} = P_0$	$P_8 = 20\,bar$	
= 1.01325 bar	$T_8 = 298.15\,K$	
Fuel: CH_4	$T_{8P} = T_9 - 15\,K$	
$H_u = 50000\,kJ/kg$	$P_9 = 20\,bar\,(sat.)$	

TABLE II. COORDINATES OF THE OPTIMUM POINTS OF FIGURE 5.

Cycle	Perfect gas		Ideal Gas	
	r*	η*	r*	η*
Simple	27	0.3457	37	0.3841
Regenerative	7	0.4045	6	0.4574
Intercooled Regenerative	13	0.4497	11	0.4918

TABLE III. COORDINATES OF THE OPTIMUM POINTS OF FIGURE 6.

Cycle	Perfect gas		Ideal Gas	
	r*	w*	r*	w*
Simple	12	344.64	14	398.04
Regenerative	14	328.93	18	372.50
Intercooled Regenerative	42	442.85	63	513.06

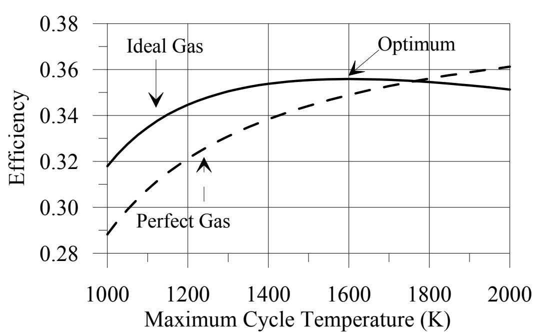

Figure 4. Efficiency of System I as a function of turbine inlet temperature with two gas models: perfect gas, ideal gas; $\eta_C = 0.90$, $\eta_T = 0.92$, *r=10.*

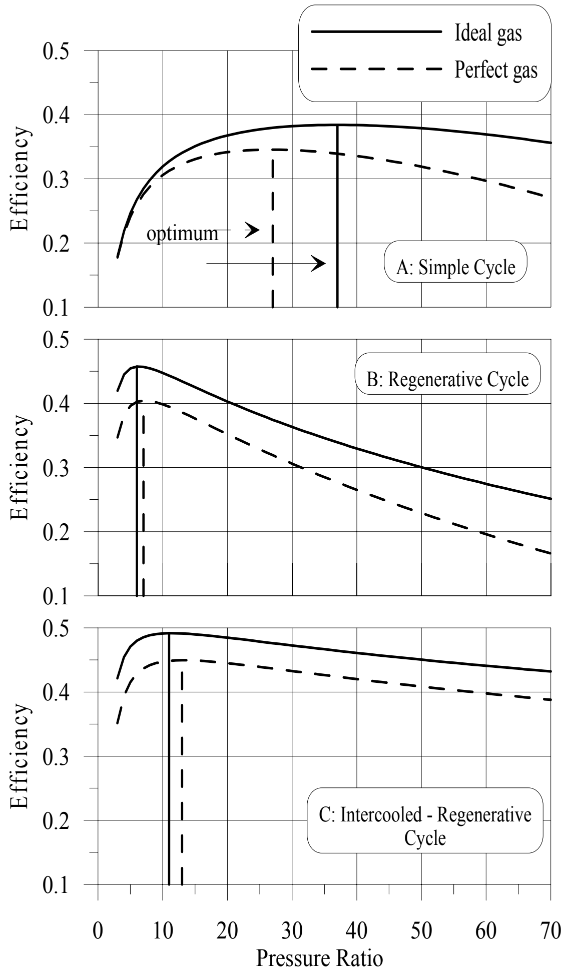

Figure 5. Efficiency as a function of pressure ratio with two gas models: perfect gas, ideal gas; η_C *=0.85,* η_T *=0.88,* τ_{max} *=5.*

A change from the perfect to the ideal gas model changes the pressure ratio for maximum efficiency by +37.04%, –14.29% and –15.38% for the simple cycle, the regenerative cycle and the intercooled-regerenerative cycle, respectively. The optimum efficiency increases by 11.11%, 13.08% and 10.10%, respectively.

A change from the perfect to the ideal gas model increases the pressure ratio for maximum specific work by 16.67%, 28.57% and 50.00% for the simple cycle, the regenerative cycle and the intercooled-regerenerative cycle, respectively. The optimum specific work increases by 15.49%, 13.25% and 15.85%, respectively.

It is noted that for *Figures 5 and 6*, component efficiencies more or less realistic have been used. For those values of efficiencies, the temperature of maximum cycle efficiency increases to an extremely unrealistic value. In order to keep the maximum point in a realistic temperature, we have considered more optimistic component efficiencies in *Figure 4*.

5. Optimization of Systems with Alternative Methods for Evaluation of Properties

Three distinct optimization problems have been formulated and solved for each system:

(a) Maximization of the cycle efficiency:

$$\max \eta_i, \qquad i = I, II, III \qquad (24)$$

(b) Maximization of the specific work:

$$\max w_i, \qquad i = I, II, III \qquad (25)$$

(c) Minimization of the annualized cost rate:

$$\min Z_i, \qquad i = I, II, III \qquad (26)$$

$$Z = FCR \cdot \phi \cdot \sum_n C_n + c_f \cdot \dot{m}_f \cdot H_u \cdot t \qquad n = C, B, T, I, X, R \qquad (27)$$

For the air standard cycle mentioned in the Introduction, the first two optimization problems have a well-known closed-form analytic solution (Guha 2003 , Frangopoulos 1988 and 1992). Changing the model from constant to variable quantity and quality and simultaneously introducing pressure losses make the analytic solution very difficult or impossible. An analytic solution for certain cases based on various simplifying assumptions has been attempted in Horlock et al. (2000) but, as mentioned in Guha (2003), the errors introduced by these assumptions may be critical. Therefore, the optimization problems are solved numerically here.

5.1 Optimization of system I

The following independent variables have been considered for the three optimization problems:

$$\mathbf{x}_{I,\eta} = (r), \quad \mathbf{x}_{I,w} = (r), \quad \mathbf{x}_{I,Z} = (\eta_C, r, \tau_3, \eta_T) \qquad (28)$$

The problems (a) and (b) have been solved for the thermoeconomic optimum values of η_C^*, τ_3^*, and η_T^* obtained from the solution of problem (c). The results are given in TABLE IV, where the optimum values of the objective functions are written in bold numbers.

The most significant effects of the change from perfect to ideal gas are the following: The optimum pressure ratio for the problems (a), (b) and (c) increases by 34.48%, 9.69% and 21.05%, respectively. The optimum efficiency of problem (a) increases by 9.85%. The optimum specific work of problem (b) increases by 13.58%. The optimum annualized cost rate of problem (c) decreases by 7.78%.

5.2 Optimization of system II

The following independent variables have been considered for the three optimization problems:

$$\mathbf{x}_{II,\eta} = (r), \quad \mathbf{x}_{II,w} = (r), \quad \mathbf{x}_{II,Z} = (\eta_C, r, \tau_3, \eta_T, \varepsilon_X) \tag{29}$$

It is noted that the useful heat rate $\dot{Q}$ is fixed; consequently, maximization of η_{II} is equivalent to maximization of $\eta_{II,tot}$.

5.3 Optimization of system III

The following independent variables have been considered for the three optimization problems:

$$\mathbf{x}_{II,\eta} = (r, \varepsilon_X), \quad \mathbf{x}_{II,w} = (r, \varepsilon_X), \quad \mathbf{x}_{II,Z} = (\eta_C, r, \tau_6, \eta_T, \varepsilon_X) \tag{30}$$

The problems (a) and (b) have been solved for the thermoeconomic optimum values of η_C^*, τ_6^* and η_T^* obtained from the solution of problem (c).

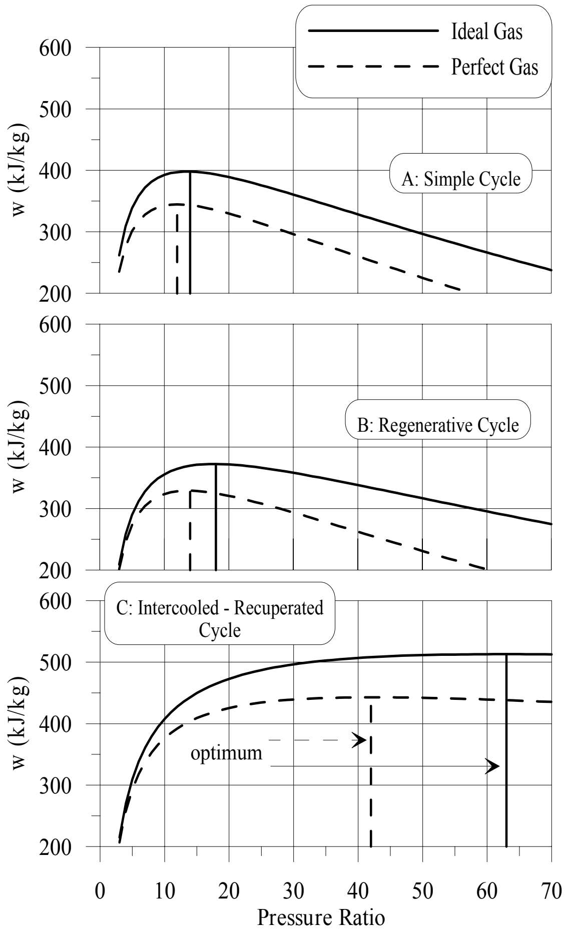

Figure 6. Specific work as a function of pressure ratio with two gas models: perfect gas, ideal gas; η_C =0.85, η_T =0.88, τ_{max} =5.

The results are given in TABLE VI. The most significant effects of the change from perfect to ideal gas are the following: The optimum pressure ratio for the problems (a), (b) and (c) changes by –44.67%, +19.47% and –24.98%, respectively. The optimum efficiency of problem (a) increases by 12.24%. The optimum specific work of problem (b) increases by 15.89%. The optimum annualized cost rate of problem (c) decreases by 9.57%.

It is worth noting also that the maximization of the specific work results in elimination of the air pre-heater (ε_x=0) in both cases (perfect and ideal gas).

5.4 Further remarks on system optimization

According to the analysis of Section 4, the optimization problem (a) of System I might be considered as having two independent variables:

$$\mathbf{x}_{I,\eta} = (r, \tau_3) \tag{31}$$

The solution of this problem gives extremely and unrealistically high optimum values r* and τ_3^* for the values of η_C and η_T considered here. This is why only the pressure ratio has been considered as an independent variable, while τ_3 is a parameter.

The effectiveness ε_x of the air pre-heater could be an independent variable for problems (a) and (b) of System II also. In such a case, maximization of the specific work would result in elimination of the air-pre-heater, changing the structure of the system. In order to keep the structure the same as in the CGAM problem, it was decided to treat ε_x as a fixed parameter for these problems.

It is interesting to note that, going from the simple cycle to the intercooled regenerative cycle, the optimum value of the annualized cost rate decreases by 25.22% (from $9.914{\cdot}10^6$\$ to $7.417{\cdot}10^6$\$), in spite of the fact that the system becomes more complex. The most important reasons for this decrease are the significant decrease of the pressure ratio (which decreases the capital cost of certain components) and the significant increase of the system efficiency (which decreases the fuel cost).

Conclusion

A preliminary performance evaluation followed by the solution of three optimization problems for each one of three different gas turbine system configurations has demonstrated that a change from the perfect gas to the ideal gas model for evaluation of properties has a very significant effect on the results, which cannot be ignored. With the computing capabilities of today, the necessary calculations are conveniently performed.

TABLE 4. OPTIMIZATION RESULTS FOR SYSTEM I (SIMPLE CYCLE).

	Perfect gas			Ideal gas		
	max η_I	max w_I	min Z_I	max η_I	max w_I	min Z_I
η_C^*	0.8445	0.8445	0.8445	0.8360	0.8360	0.8360
r*	29.0974	12.3112	18.9046	39.1313	13.5037	22.8845
τ_3^*	4.9924	4.9924	4.9924	4.9887	4.9887	4.9887
η_T^*	0.9047	0.9047	0.9047	0.9048	0.9048	0.9048
η_I	**0.3686**	0.3686	0.3686	**0.4049**	0.3582	0.3915
w_I	312.98	**356.44**	345.77	336.15	**404.84**	387.97
Z_I	$1.120 \cdot 10^7$	$1.110 \cdot 10^7$	$\mathbf{1.075 \cdot 10^7}$	$1.055 \cdot 10^7$	$1.037 \cdot 10^7$	$\mathbf{9.914 \cdot 10^6}$

TABLE 5. OPTIMIZATION RESULTS FOR SYSTEM II (COGENERATION SYSTEM).

	Perfect gas			Ideal gas		
	max η_{II}	max w_{II}	min Z_{II}	max η_{II}	max w_{II}	min Z_{II}
η_C^*	0.8408	0.8408	0.8445	0.8323	0.8323	0.8323
r*	9.3053	13.4909	9.2207	12.3133	16.3740	10.4584
τ_4^*	5.0271	5.0271	5.0271	5.0292	5.0292	5.0292
η_T^*	0.8876	0.8876	0.8876	0.8864	0.8864	0.8864
ε_X	0.7675	0.7675	0.7675	0.6683	0.6683	0.6683
η_{II}	**0.3792**	0.3719	0.3791	**0.3972**	0.3933	0.3959
w_{II}	322.95	**330.79**	322.55	367.41	**371.84**	360.87
Z_{II}	$1.002 \cdot 10^7$	$1.029 \cdot 10^7$	$\mathbf{1.002 \cdot 10^7}$	$9.577 \cdot 10^6$	$9.734 \cdot 10^6$	$\mathbf{9.541 \cdot 10^6}$

TABLE 6. OPTIMIZATION RESULTS FOR SYSTEM III (INTERCOOLED REGENERATIVE CYCLE).

	Perfect gas			Ideal gas		
	max η_{III}	max w_{III}	min Z_{III}	max η_{III}	max w_{III}	min Z_{III}
η_C^*	0.8579	0.8579	0.8579	0.8612	0.8612	0.8612
r*	9.8729	41.2845	8.5835	5.4628	49.3242	6.4390
τ_6^*	5.0213	5.0213	5.0213	5.0044	5.0044	5.0044
η_T^*	0.8919	0.8919	0.8919	0.8878	0.8878	0.8878
ε_X	0.9602	0.0	0.9630	0.9711	0.0	0.9691
η_{III}	**0.4729**	0.3813	0.4724	**0.5308**	0.4102	0.530
w_{III}	383.55	**458.00**	322.55	325.34	**530.79**	351.03
Z_{III}	$8.218 \cdot 10^6$	$1.116 \cdot 10^7$	$\mathbf{8.202 \cdot 10^6}$	$7.433 \cdot 10^6$	$1.077 \cdot 10^7$	$\mathbf{7.417 \cdot 10^6}$

Nomenclature

C_n	capital cost of component n (installed)
c_f	unit cost of fuel
c_p	specific heat capacity at constant pressure
$\tilde{c}_p$	molar heat capacity at constant pressure
c_v	specific heat capacity at constant volume
$\tilde{c}_v$	solar heat capacity at constant volume
FCR	fixed charge rate
f	fuel to air ratio: $f = \dot{m}_f / \dot{m}_a$
H_u	lower heating value of the fuel
k	specific heat ratio: $k = c_p / c_v$
$\dot{m}_a$	air mass flow rate
$\dot{m}_f$	fuel mass flow rate
$\dot{m}_g$	exhaust gas mass flow rate
P	pressure
r	pressure ratio
$\dot{Q}$	useful heat rate of the cogeneration system (production of steam)
$\tilde{R}$	universal gas constant
T	absolute temperature [K]
T_1	compressor inlet temperature
T_3	turbine inlet temperature (simple cycle)
t	period of operation during a year
$\dot{W}$	net power to the load
w	specific work, as defined by Eq. (17)
x	set of independent variables for optimization
x_i	molar fraction of species i in a mixture
Z	annualized cost rate of a system, in $ (including capital as well as operation and maintenance expenses)

Greek letters

γ	$\gamma = (k-1)/k$
η	efficiency
η_B	efficiency of the combustor
η_C	isentropic efficiency of the compressor
η_J	efficiency of the Joule cycle
η_m	mechanical efficiency
η_T	isentropic efficiency of the turbine
τ_i	temperature ratio: $\tau_i = T_i / T_1$
ϕ	maintenance factor

Subscripts

A	air standard gas turbine cycle
a	air
B	combustor
C	compressor
f	fuel
g	exhaust gases
I	intercooler
T	turbine
R	exhaust gas boiler
X	air preheater
0	standard conditions: 25°C, 1.01325 bar

Superscripts

*	optimum value

References

Frangopoulos C.A., 1988, "Optimal Design of a Gas Turbine Plant by a Thermoeconomic Approach", In: 2nd International Symposium and Exposition on Turbo-machinery, Combined-Cycle Technologies and Cogeneration, Montreaux, Switzerland.

Frangopoulos C.A., 1992, "Thermoeconomic versus thermodynamic design optimization of a simple gas turbine plan. Part A′: Thermodynamic optimization. Part B′: Thermoeconomic optimization" *The International Journal of Mechanical Engineering Education;* 20(3):149–168.

Frangopoulos, C.A., 1994, "Application of the Thermoeconomic Functional Approach to the CGAM Problem" *Energy*; 19(3): 323-342.

Gordon S., McBride B.J., 1994, "Computer program for calculation of complex chemical equilibrium compositions and applications; I. Analysis" NASA reference publication 1311, National Aeronautics and Space Administration.

Guha A., 2003, "Effects of internal combustion and non-perfect gas properties on the optimum performance of gas turbine", *Proceedings of the Institute of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science;* 217: 1085-1099.

Haywood R.W, 1987, *Analysis of engineering cycles.* 3rd ed. Oxford: Pergamon Press.

Horlock J.H., 2003, *Advanced gas turbine cycles.* Amsterdam: Pergamon.

Horlock J.H., Woods W.A., 2000, "Determination of the optimum performance of gas turbines", *Proceedings of the Institute of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*; 214: 243-255.

Horlock J.H., 2001, *Erratum* (for Horlock 2000). *Proceedings of the Institute of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science;* 215: 1378.

McBride B.J., Zehe M.J., 2002, "Gordon S. NASA Glenn coefficients for calculating thermodynamic properties of individual species" NASA/TP–2002-211556, National Aeronautics and Space Administration.

Valero A. et al., 1994, "CGAM Problem: Definition and conventional solution" *Energy*; 19(3): 279-286.