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Procedures for the Search of the Optimal Configuration of District Heating Networks

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Abstract

This paper deals with the choice of the optimal configuration of the district heating network to be built in an urban area. The users to be connected with the network are determined so that an economic objective function is optimized. In this approach, the average unit cost of heat is considered as the function to be minimized. An alternative heating system is considered for the users not connected with the network.

In this work, three different iterative procedures are presented. All these procedures start with an initial superstructure connecting the possible users. The initial structure is progressively simplified by disconnecting one user at each iteration. The three procedures differ in the algorithm for the network simplification: the first procedure is deterministic, while the others use probabilistic approaches derived from the simulated annealing technique.

The procedures are applied to a small portion of the urban tissue of Turin and their effectiveness is compared.

Keywords: District heating network, optimal configuration, simulated annealing, thermoeconomics

1. Introduction

District heating (DH) is a rational way of providing heat to multiple users. The system is constituted by one or a few centralized plants, generally cogeneration plants (CHP) and high efficiency boilers, which feed the network. In this way it is possible to reduce the emission of air pollutants and the resource depletion with respect to private thermal generation.

The main issue on a district heating system refers to its cost. As with any other system it needs to be more convenient than the alternatives. A second issue must be considered: building these systems requires several years and produces lots of problems for the municipality. For this reason, the system must be designed in its final structure, with few possibilities for making changes. In particular, the design involves the option for the possible users to be connected, the topology and the pipe diameter of each branch.

Such a problem can be solved as a synthesis problem, i.e. an optimization where the system structure is not defined a priori (Frangopoulos et al., 2002). In this way it is possible to search for the optimal network that minimizes (or maximizes) an objective function, such as the minimum cost of heat or the maximum benefit.

This paper aims to propose a thermoeconomic approach for determining the optimal network extension (which users are connected and the number) in terms of general cost - monetary, environmental, etc.

The theoretical considerations are applied to a network, whose possible users are constituted by the buildings located in an area in the west part of Turin, close to the area at the moment actually connected with the district heating network (DHN). The thermal plant is considered to be in the center of this area.

The thermal plant is assumed to be constituted by a cogeneration combined cycle and some boilers. The CHP is designed to provide 40% of the maximum thermal load, which corresponds to more than 70% of the annual thermal energy request. Cogeneration is

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*Author to whom correspondence should be addressed obtained through a steam extraction at about 2.5 bars from the turbine, which feeds a heat exchanger. The remaining requests are covered by means of boilers (Verda, 2001). These percent values are assumed to be independent of the number of users and thus of the total load. Such an assumption is made since the thermal plant design is performed after the determination of the optimal network extension.

The users are grouped together, using geographic criteria, into zones, represented on the network map as single points called thermal barycenter (TB). A tree-shaped sub-network connects each single user of the area to the corresponding thermal barycenter. The DHN is a closed network; the water temperature in the outgoing pipes is assumed to be at about 120°C, while in the return pipes is assumed to be at about 60°C, with load variations mainly controlled by operating on the water mass flow rate. A heat exchanger located in each building operates the connection between the main network and the building distribution system.

Water circulation through the network is obtained by means of several pumps, generally located corresponding to the main ramifications.

2. System Model

In this paper the optimal configuration of the DH network is approached by starting with a superstructure (Frangopoulos et al., 2002), which is a network connecting all the possible zones and thus all the users. The technique is widely used for solving synthesis problems (see for example Frangopoulos and Dimopoulos, 2004, Rancruel and von Spakovsky, 2004, Li et al. 2004). Once the superstructure is built, the problem can be solved as an optimization problem; particular values of the variables associated with the components (e.g. the internal flows) correspond to the condition of absence of that component. In the case of DH systems, when the optimal mass flow rate in a pipe is zero, the pipe must be eliminated from the structure.

The procedure begins with the evaluation of the objective function in the initial condition, corresponding to all the users connected with the network. The network is then simplified by disconnecting some of the users. The simplification is based on thermoeconomic criteria in the first procedure and thermoeconomic criteria combined with a probabilistic approach in the other procedures herein proposed. Thermoeconomics is used with the aim of disconnecting the users that determine high costs and eliminating the pipes connecting these users with the rest of the network. For the disconnected users heating is obtained with a convenient alternative that, in this paper, is assumed for simplicity's sake as a local boiler. The procedure is stopped when all the users are disconnected; it is not safe to stop the procedure when a minimum/maximum is reached because of the possible presence of local minima/maxima.

The details of the different procedures are shown by considering the objective function to be minimized, the average unit cost of heat provided to the users. The same procedures can be applied by considering a different objective function, such as the minimum exergy consumption. In this case the variables for the network simplification should be changed accordingly, i.e. the economic unit cost could be substituted with the exergetic unit cost.

The first step consists of calculating the average unit cost of heat as:

$$\bar{c} = \frac{C_{tot}}{Q_u} = \frac{C_{net} + c_F \cdot Q_F + c_P \cdot L_p}{Q_u} \qquad (1)$$

The cost of network C_{net} includes the purchase of insulated pipes, heat exchangers, pumps and valves, together with other direct costs, such as excavation, installation and paving restoration. Indirect costs, such as engineering, legal costs, contingency, insurance, as well as additional costs, are also included; these costs have been estimated on the basis of the purchasing costs (Bejan et al., 1996). Maintenance is also included.

C_{net} is an annual cost. Year is the best unit time to be used for the techno-economic (or thermoeconomic) analysis of such a system due to the variation in the production depending on the average external temperature during the day. This cost is calculated as an ordinary annuity of the total investment cost, which depends on the total life of the plant and the rate of return.

The unit cost of heat exchanged between the thermal plant and the DH network has been calculated starting from the electricity cost of a non-cogeneration combined cycle and then assigning the cost of non-produced electricity to the heat. This occurs because the steam extraction reduces the electricity production. Moreover, the unit cost of electricity has been considered as linearly dependent on the plant size.

This procedure for the calculation of the unit cost of heat is not general; a thermoeconomic analysis of the power plant could provide more rational results.

The heat requested by the users Q_u and the heat supplied by the thermal plant Q_F differ because of heat losses. This difference has been evaluated as 6% of the heat request during a year. The losses have been internalized in the unit cost of heat supplied to the network so that Q_u and Q_F can be assumed coincident.

The last term at numerator of equation (1) accounts for the electricity cost for pumping, with c_P the unit cost of electricity and L_p the annual electricity consumption, calculated as:

$$L_p = \frac{1}{\eta_p} \int_{\text{year}} G \cdot v \cdot \Delta p \cdot dt \qquad (2)$$

where η_p is the average pump efficiency, G is the water mass flow rate, v is the water specific volume (assumed constant) and Δp the total pressure losses due to pipe friction and localized resistances.

The terms in equation (1) depend on the thermal load supplied by the network and on its extension. The first step of the analysis consists in selecting the localization of the thermal plant, generally subjected to multiple constraints, technical, social, etc. The possible area to be heated by the thermal plant must be chosen as well. This area can be divided into zones, each including one or more buildings. The number of zones should be selected as a trade off between result accuracy (large number of zones) and time required for design and calculation (small number of zones). For each zone, the total volume of buildings is determined. The thermal barycenter can be easily located in the area by considering the position of buildings and their respective volume (the geometric barycenter can be used as well, especially when the building structure is sufficiently regular). At this point, the network connecting the thermal plant with TBs can be traced. In the case herein analyzed, the initial superstructure is constituted of a total volume of buildings equal to $25 \cdot 10^6$ m^3. This area has been divided into 72 zones, connected with the thermal plant as shown in *Figure 1.*

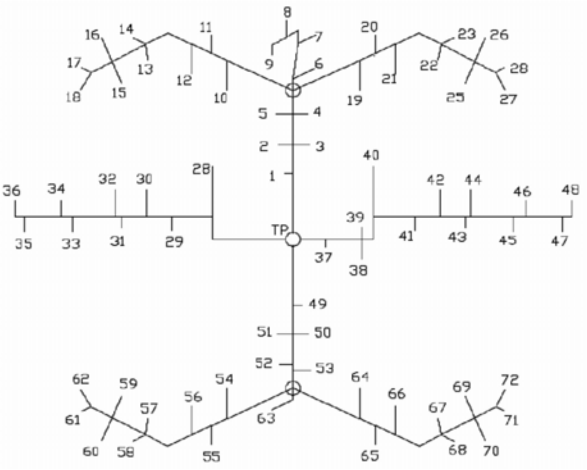


Figure 1. Sketch of the network with barycentric thermal plant.

The annual heat load of each single zone Q_z is calculated by considering, for the whole heating season, the daily difference between the internal temperature (20°C) and the external temperature, the average thermal transmittance of buildings (through walls, windows, floors, etc.) and the number of daily heating hours (hh). The thermal transmittance of buildings can be multiplied for a shape factor defined as the ratio of the external surface and building volume; this quantity, indicated as r, expresses the volumetric heat losses per unit temperature difference. Its value has been experimentally calculated by the authors for almost 20 buildings connected with the district heating network. For this goal, the thermal request, the internal and external temperatures have been measured each month of the year. An average value of 0.9 $W/(m^3K)$ can be assumed, since experimental data have provided values for this parameter in the range between 0.87 and 0.93 $W/(m^3K)$, with small variations during the year. The annual heat load for a zone in kWh is then expressed as:

$$Q_z = \frac{r \cdot DTD \cdot hh \cdot V_z}{1000} \qquad (3)$$

where V_z is the total volume of buildings in the zone and DTD is the summation of the daily difference between the internal and external temperature, calculated for the whole heating season. This value is available for all the Italian municipalities or can be easily calculated from historical data. The number of daily heating hours, hh, is established by law, depending on DTD. For the town of Turin, DTD is 2014°C day/year, the heating season from the middle of October to the middle of April, while the number of heating hours is 12 per day.

The total heat load is calculated as the summation of the contributions of all the zones. The network operates longer than specified, mainly due to four causes: 1) non-contemporary request by the users, 2) presence of particular users, like hospitals, that require heat for more than 14 hours per day and for an extended period, 3) requirement of domestic water, 4) presence of users that require heat in summer for air conditioning through absorption chillers. For all these reasons, the total load calculated through equation (3) has been considered as spread on 18 hours per day in the seasonal heating, moreover the thermal flow outside this period has been assumed non-null, but 11% of the maximum thermal flow. This last assumption has been formulated by making use of the actual heat load curves.

The purchase cost of the DHN is calculated by considering the contributions of the insulated pipes constituting the main network (from the thermal plant to each thermal barycenter), the insulated pipes of each sub-network (from the barycenter to the buildings on the zone), the pumps, the special components, such as valves and junctions between pipes, the heat exchangers in the buildings and in the thermal plant.

The purchase cost of the insulated pipes is expressed through a polynomial function, obtained by interpolating available data:

$$PC_{IP} = \left(a_0 + a_1 \cdot D_{int} + a_2 \cdot D_{int}^2\right) \cdot 1.25 \cdot L \cdot 2 \qquad (4)$$

where D_{int} is the internal diameter and L the length of the considered trait, 1.25 is a corrective factor used to include the cost of special components also determined through available data and 2 accounts for the double pipe. The calculated values of polynomial coefficients are: a_0=6.86 €/m, a_1=0.31 €/(mm·m), $a_2=0.4\cdot10^{-3}$ €/(mm^2·m).

The internal diameter is calculated by first determining the mass flow rate in each branch. The mass flow rate is imposed by the thermal requirement of each user downstream from that branch:

$$\Phi = G \cdot \left(h_o - h_r\right) \qquad (5)$$

where Φ is the thermal flow provided to the users (the maximum load is considered in design), G the water mass flow rate, h_0 and h_r the enthalpies of fluid feeding the users and returning from the users. The diameter is determined by imposing the maximum velocity v_{max} allowed in the pipes. This value is mainly defined on the basis of economic criteria, since friction losses and thus pumping cost depend on the square of velocity. On the other hand, a too low velocity would determine a large pipe diameter, thus high investment costs. In this analysis a value of 2.5 m/s is considered. The water mass flow rate G is expressed as:

$$G = \rho \frac{\pi D_{int}^2}{4} v_{max} \qquad (6)$$

The sub-network length is calculated by using an empirical formulation obtained by interpolating five hundred cases randomly generated by varying the number of buildings on a rectangular shaped zone and its area (Verda et al., 2004). The sub-network length L_{SN} as a function of the zone area A_Z and the number of buildings U is assumed:

$$L_{SN} = b_1 + b_2 \cdot \sqrt{A_Z} + b_3 \cdot U \qquad (7)$$

where b_1=-2767.21 m, b_2=7.796 m/m, b_3=52.404 m.

The purchase cost of the heat exchangers has been calculated as the function of the heat transfer area, according with a general function (Bejan et al., 1996):

$$PC_i = PC_0 \cdot \left(\frac{X_i}{X_0}\right)^{\alpha} \qquad (8)$$

where PC_0 is the known cost of the device at a specific size, X is a variable selected for expressing the component size, X_i is its value for the device whose cost is calculated and X_0 its reference value. For heat exchangers the variable expressing the component size is the heat transfer area. Reference values PC_0 and X_0 are respectively assumed to be 187 € and 0.65 m^2, while α=0.66.

A similar expression is considered for pumps. The electric power is considered to be the quantity for expressing the component size. Reference values PC_0 and X_0 are respectively assumed to be 35000 € and 135 kW, while α=0.65.

The costs for installation have been calculated by determining the dimensions of the excavation. It has to be 500 mm wider than the external diameter and 650 mm deeper; sustaining and covering layers of sand 100 mm high is also required. The specific costs of sand, excavation work and paving restoration (material and work) have been assumed respectively 18 €/m^3, 5.16 €/m^3 and 10.33 €/m^2. An example of excavation is shown in *Figure 2*.

The costs for the control system and the insurance during the network construction and the engineering, legal and contingency costs are calculated as percentages of the direct costs. The assumptions are shown in TABLE I. In TABLE II, the maintenance and insurance costs for the DHN, expressed as percentages of the capital costs, are shown.

The maintenance cost of the heat exchangers has been considered to be 5% of the purchase cost.

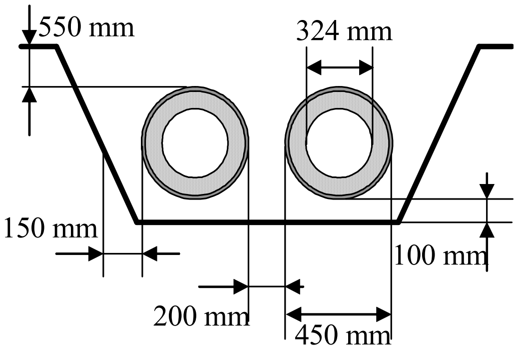


Figure 2. Schematic of excavation for the network installation.

TABLE I. INDIRECT COSTS AND CONTROL SYSTEM COST AS PERCENTAGES OF THE DIRECT COSTS.

Capital costs	Percentage
Control system	2 %
Engineering	8.2%
Legal	1.2%
Investment insurance	1.2%
Contingency	7.4%

TABLE II. OPERATIONAL COSTS AS PERCENTAGES OF THE CAPITAL COSTS.

Operational costs	Percentage
Maintenance	3%
Plant insurance	0.2%

Both capital and operational costs have been amortized. For the first ones a discount rate of 6% has been considered. The equivalent annual cost has been computed as:

$$C_C = Z_{tot} \frac{(1+d)^l}{(1+d)^l - 1} \cdot d \tag{9}$$

in which Z_{tot} is the capital cost, d is the discount rate and l is the life of the network, expressed in years.

When considering the operational costs, that are paid each year, an inflation rate has to be involved in the economic analysis. In this paper the value of the inflation rate is assumed to be 2.5% and the operational costs are calculated as:

$$C_{OP} = P_0 \cdot \frac{k \cdot \left(1 - k^l\right)}{1-k} \cdot \frac{d \cdot (1+d)^l}{(1+i)^l - 1} \tag{10}$$

$$k = \frac{1+i}{1+d} \tag{11}$$

P_0 is the annual operational cost estimated at the beginning of the first year. Equation (10) allows calculating a constant annuity all along the life of the system, taking into account that the cost P increases each year due to inflation.

The overall equivalent annual cost of the asset is given by the summation of equivalent annuity for capital and operational expenses, namely:

$$C_{net} = C_C + C_{OP} \tag{12}$$

The average unit cost of heat has been then calculated by using equation (1).

3. Thermoeconomic Analysis

A thermoeconomic analysis is implemented for the designed network, where all the possible users are connected. In particular, a useful approach that can be adopted for this purpose is the one proposed by Valero and co-workers in the eighties (Lozano and Valero, 1993 and Valero et al., 1986). One of its main characteristics is the matrix based approach, in particular the use of an incidence matrix for expressing the equation of cost conservation. The concept of the incidence matrix (see for example Chandrashekar, Wong, 1982) was formulated in the ambit of the graph theory (Harary, 1995), which is widely adopted for the topology definition as well as the fluid dynamic and thermal calculation of distribution networks (Calì, Borchiellini, 2002). The incidence matrix, **A**, is characterized by as many rows as the branches (m) and as many columns as the nodes (n). The general element A_{ij} is equal to 1 or –1, respectively if the branch *j* is entering or exiting the node *i* and 0 in the other cases. The use of the incidence matrix allows for expressing the balance equation of the flow of the general extensive quantity G_x as:

$$\mathbf{A} \cdot \mathbf{G_x} + \mathbf{G_{xd}} = \mathbf{0} \tag{13}$$

where $\mathbf{G}_x$ is the vector containing the values assumed by the quantity $\mathbf{G}_x$ in the nodes and $\mathbf{G}_{xd}$ is the vector that allows accounting for the amount destroyed in the branches, if it is not null. In thermoeconomics, equation (9) allows for the writing of the cost balance as:

$$\mathbf{A} \cdot \mathbf{\Pi} + \mathbf{Z} = \mathbf{0} \tag{14}$$

where $\mathbf{\Pi}$ is the vector containing the cost of all the flows, while **Z** contains the cost rate of the components. The calculation of all the costs requires the formulation of n-m auxiliary equations, which are obtained through a definition of resources and products of each component, expressed in terms of exergy flows (Tsatsaronis, Winhold, 1985). The auxiliary equations were formulated as four propositions, the first of which (P1) is the conservation of cost, expressed by equation (14) (Lozano and Valero, 1993). The others are: (P2) in the absence of a different evaluation, the economic unit cost of an exergy flow entering the system from the environment can be assumed to be equal to its price; (P3) in the absence of a different evaluation, the unit cost of a lost exergy flow is the same; (P4a) if the fuel of a component is defined as the difference between two exergy flows, the unit cost of these flows is equal; (P4b) if the product of a component is defined as the summation of two or more flows, the unit cost of these flows is the same.

In the case of a DH network, an auxiliary equation to be widely applied is the assignment of the same unit cost to the flow exiting each bifurcation (Verda et al., 2001).

The unit cost of a flow c can then be calculated by dividing the costs for the corresponding exergy flow:

$$c = \frac{\Pi}{\Psi} \cdot 3600 \qquad (15)$$

where Π is the thermoeconomic cost of the flow and Ψ its exergy.

At this point, the unit cost for each user can be calculated. This cost is not the same for all the users because of the different exergy destruction (mainly due to friction) and the pipe cost associated with the different paths joining the thermal plant with the users.

4. A Deterministic Thermoeconomic Approach

In this section, the optimization procedure proposed in a former paper (Verda et al., 2004) is presented. A flow chart of this procedure, indicated here as the TE procedure, is shown in *Figure 3*. The procedure starts with an initial network configuration. This configuration is a tree-shaped network connecting all the γ users. Then an iterative simplification of the network structure is performed. This is composed of the following steps: 1) network design, which mainly consists of determining the diameter of each stretch and, consequently the cost of the total network through equations (4)-(9); 2) calculation of the average unit cost of heat supplied to the connected users by means of equation (1); 3) in case the unit cost of heat in the current configuration is lower than the previously analyzed configurations, storage of this configuration; 4) calculation of the unit cost of heat for each single user; and 5) determination of a new configuration by disconnecting the user characterized by the highest value of the unit cost of heat. The iterative process is repeated until all users are disconnected.

The main advantage of such a procedure is that large structures characterized by hundreds of users can be easily processed; the computational time is linearly dependent on the number of decision variables, i.e. the number of users. In contrast, a disadvantage is that it does not guarantee the obtainment of the true optimum. In particular, this event can occur when several users in different zones of the network are characterized by values of the unit costs close to the unit cost of the user that is highlighted as the one to be eliminated. In such a case, the procedure formerly proposed allows one to obtain a quasi-optimal structure, although often just slightly different from the true optimal structure. In the following section, the results obtained with this method are briefly presented.

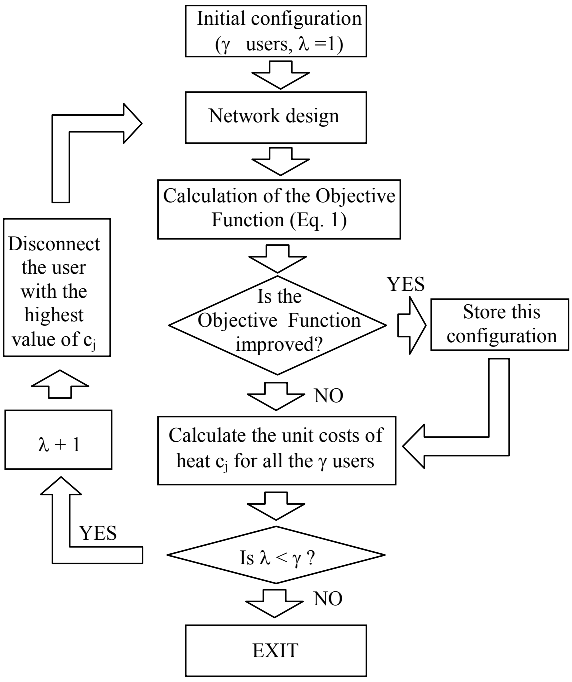


Figure 3. Flow chart of the TE procedure.

5. Application of the TE Approach to a DH Network

The procedure described above has been applied to the DH network shown in *Figure 1*. The number of buildings in each area varies in the range between 5 and 125, while the thermal load varies in the range between 2670 and 42060 kW. The network superstructure, sketched in *Figure 1*, has a unit cost of 0.03817 €/kWh while the total cost is 25.419 M€.

The unit cost of each zone is calculated through thermoeconomic analysis, in order to define the order for the reduction of the initial superstructure. The results, shown in *Figure 4,* highlight that the zones 71, 16 and 69 are to be disconnected since they are characterized by the highest costs. These users are characterized by high costs for pumping and investment and a high unit cost due to the low thermal need.

The main driving force for disconnecting the users is the distance from the plant: this fact causes them to have high capital costs due to the pipes, but also high pressure losses which determine high costs for pumps and pumping. This driving force is amplified for zones characterized by a smaller thermal request. Their unit cost becomes high, causing them to be deactivated.

The determined value of the minimum average unit cost of heat is 0.03769 €/kWh and it occurs for a number of zones equal to 31. In *Figure 7a* the network so found is presented. The corresponding total cost, C_{net}, is 14.218 M€/year.

6. Simulated Annealing

In this paper, two other methods are presented and applied, both iterative and based on probabilistic criteria for activating or deactivating the users. The idea at the basis of the two adopted methods is known as simulated annealing (SA).

The simulated annealing approach to solving optimization problems stems from the procedure used to harden steel. First, steel is heated to a high temperature close to the transition to the liquid phase. Then it is cooled more or less slowly: this is the annealing. In order to achieve a low free energy configuration, the cooling speed must be low enough so that the molecules have time to find position in an ordered pattern. If the cooling is too fast, the configuration might correspond to a local minimum and not to a global one.

In SA the correspondance between macro variables is statistically described and the interactions between particles are not taken into account. The first to formulate a probabililstic law linking the temperature to the frequencies of the many possible energy states is Boltzmann. Other scientists and researchers have been using this law to simulate the annealing.

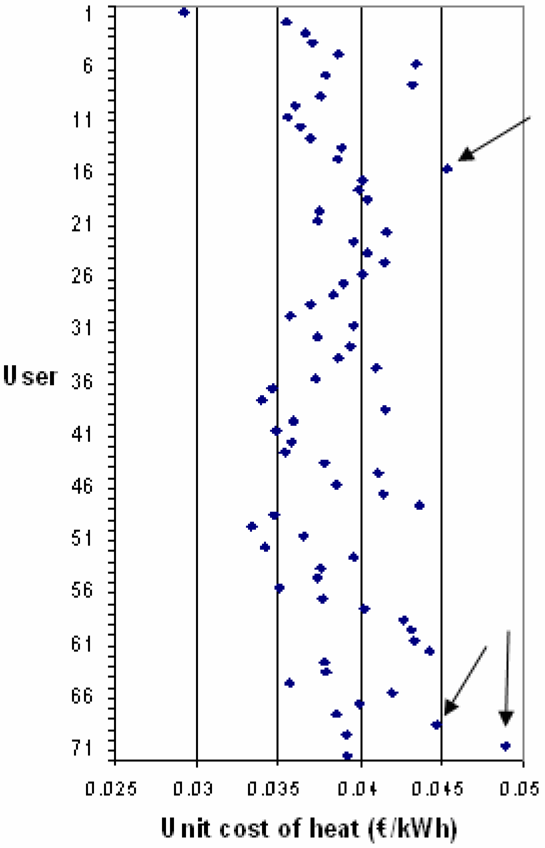


Figure 4. Unit thermoeconomic cost calculated for all the zones.

In the simulation, energy states are randomly generated. They are compared with the former one E_{old}: if the new state, E_{new}, is lower than the old one, then E_{new} survives. Otherwise, if E_{old} is lower than E_{new}, there is still a probability for the system to leave the old state and enter the new one. This probability is computed as:

$$p = \exp\left(-\frac{E_{new} - E_{old}}{K \cdot T} \right) \qquad (16)$$

where K is the Boltzmann constant and T is the current temperature of the system. T changes when it is no longer representative of the state of the system. For this change, usually a linear law is adopted. The higher is the difference between the two energy configuration, the lower is the probability for the system to enter in the new state, but still that probability is not null. Therefore, the SA procedure has the ability to escape from local optima and to reach the global one (Schwefel, 1994).

The SA method can be used to describe phenomena other than the annealing. In those cases, K and T are parameters that have to be chosen very carefully. T gives the information on the state of the system and K is a constant that influences the value of p (Metropolis, 1953).

In the following, two procedures derived from the SA technique are described. As with the TE procedure, these are iterative: At each iteration the network structure is simplified by disconnecting one user. The difference between the two procedures is the criterion adopted in order to select the user to be disconnected. Both procedures use a probabilistic approach. This means that each time the algorithm is run, it is possible that the optimal configuration differs from the result obtained in the previous run. For this reason the accuracy of this methods, i.e. the probability of finding the true optimal configuration, can be increased by running the whole procedure several times (n_{run}). In this application, the two procedures have been run 1000 times.

7. First SA Based Method

In this first method, the unit cost of all the users is calculated. Once this is done, a probability function p is assigned to each user. This represents the probability of being deactivated:

$$p_j = M \exp\left(\frac{DC_j}{K \cdot T} \right) \qquad (17)$$

The term DC_j stands for the difference between the unit cost of user j and the average cost of heat calculated at that iteration. K is fixed to a constant value, T is assumed as the number of users connected to the network and M is a term that allows one to obtain a summation of all the probabilities equal to 1. Index j is first set to 1, as the users are processed in order of identification; p is set equal to the probability p_j. A random number x is then extracted. If x is higher than p_j, the user j is deactivated; otherwise p is increased by the value p_{j+1} and j is set to j+1. The test starts again until a value of p greater than x is found.

When a zone is deactivated, a new configuration is found. Its heat average cost is calculated as well as the unit cost of all the users still connected. T is diminished to 1 and the procedure starts again. The calculation continues until all the zones are deactivated. A flow chart of this procedure is presented in *Figure 5.*

Although this procedure considerably differs from the original SA technique, it presents similar characteristics that allow overcoming the problem of local minima.

The final configuration obtained with this method, presented in *Figure 7b*, is slightly different from that previously presented and its unit cost is different too: 0.03756 €/kWh. There are 43 zones connected and C_{net} is 17.835 M€/year. This result has been obtained more than once by running the procedure 1000 times.

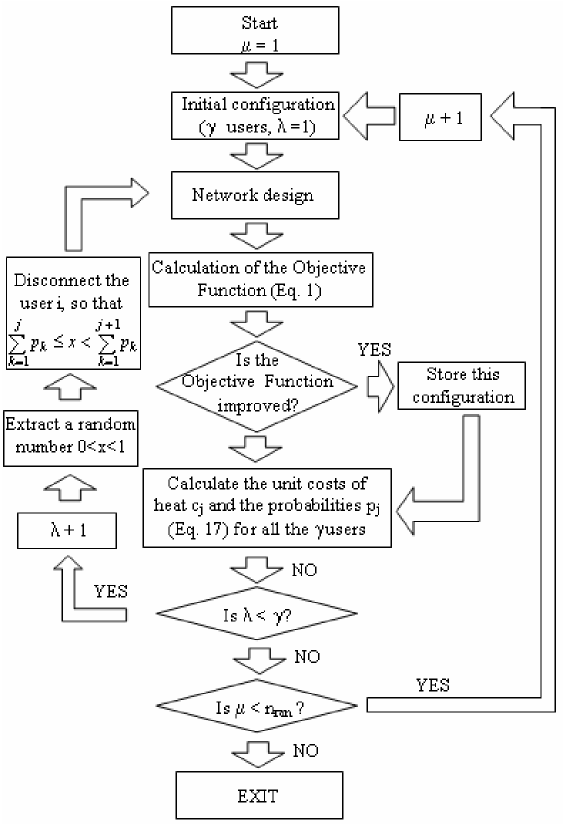


Figure 5. Flow chart of the first SA based procedure.

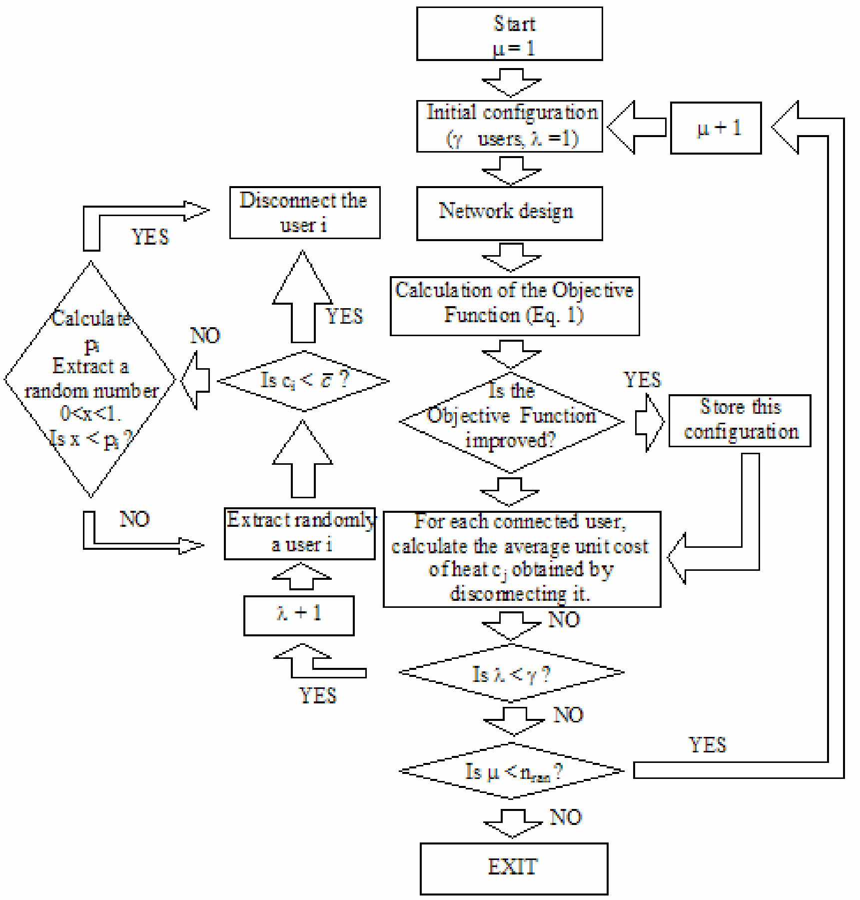
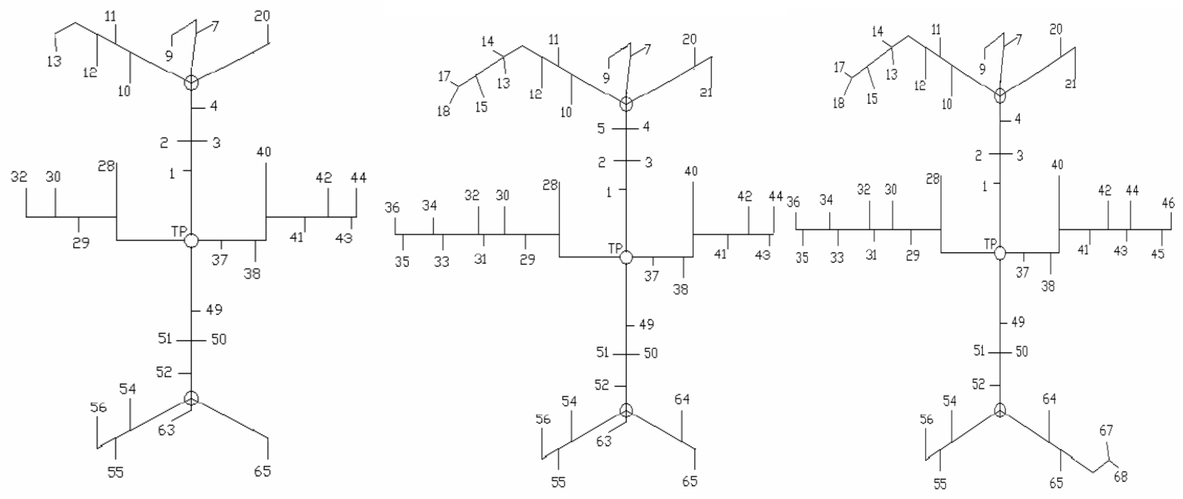


Figure 6. Flow chart of the second SA based procedure.



(a) (b) (c)

Figure 7. Best network configurations obtained with the TE method (a), the first SA based method (b) and the second SA based method (c).

8. Second SA Based Method

The second method presented here is more similar to the typical SA approach than the previous one. The method starts with all the users connected to the network. The average cost of heat is computed as previously described.

One of the active zones is randomly selected. If the average cost calculated for deactivating it is less than the average cost of the actual network, the zone is definitely deactivated; if its cost is greater, then a random number x is extracted and is compared to the probability p_j assigned to that zone. Probabilities are assigned by using the following function:

$$p_j = M \exp\left(-\frac{DCN_j}{K \cdot T}\right) \tag{18}$$

where DCN_j is the difference between the average heat cost of the network without user j and the average heat cost of the network at the previous step (i.e. the cost of the network with user j connected).

If p_j is greater than x, the configuration without j is rejected; if not, it is accepted and it becomes the reference configuration. A new user to be disconnected is selected and the procedure continues till all the users are deactivated.

With this method, the best configuration obtained after 1000 runs has 45 users connected and a unit cost of 0.037567€/kWh. The corresponding value of C_{net} is 18,510 M€/year. The final configuration obtained with this method is presented in *Figure 7c*.

9. Conclusions

In this paper three procedures for the choice of the optimal configuration of distributing network systems, such as district heating or cooling systems, are proposed. The procedures are based on the definition of an initial superstructure, connecting all the possible users. The users are grouped into zones in order to simplify the problem when large systems are considered. The procedures make use of thermoeconomic quantities as criteria for simplifying the initial superstructure and for finding the best configuration. The unit cost is here considered as an effective quantity whatever objective function is considered. In the first procedure the progressive simplification of the initial superstructure is made by disconnecting at each iteration the user characterized by the maximum unit cost. Instead, in the two other procedures, a probability distribution depending on the unit cost of each user is considered: the users characterized by a high unit cost have a low probability of being included in the determined optimal structure and vice versa. The user's disconnection is randomly operated and the overall procedure is repeated several times. For the users not connected with the network, heating is provided though local boilers. At the end, the structure that performed the best result is selected. The procedures do not guarantee obtaining the true optimum, nevertheless a quasi-optimal configuration is always found.

The procedures have been applied to the same DH network with a barycentric thermal plant. The results are summarized in TABLE III.

TABLE III. CHARACTERISTICS OF THE BEST NETWORK OBTAINED BY APPLYING THE TE, SA1 AND SA2 PROCEDURES.

Network structure	Initial	Best TE	Best SA1	Best SA2
Users	72	31	43	45
$\bar{c}$ (€/MWh)	38.17	37.69	37.55	37.56
Total cost of network (M€)	25.42	14.22	17.83	18.15

All the different configurations improve the initial structure, determining a lower average unit cost of heat for the users. The TE procedure generally does not allow the obtaining of the true optimal structure. The main problem for this procedure can occur when two or more users located in the same zone are characterized by different thermal loads, in particular when one user is characterized by thermal request much larger than the others. This situation occurs, for instance, in the case of users 28 to 36. User 32 has a larger thermal load than the others, thus its unit cost is the lowest (see *Figure 3*). The progressive elimination of users 33-36 and 31, which are located downstream of user 32 with respect to the thermal plant, seems to produce benefits. Instead, the impact on the overall optimization process is negative, mainly because of the cost C_{net} for this part of the network, which is mainly charged on user 32. This also means that each of the disconnected users has a negative impact on the average cost of heat, but if they are considered all together, they contribute to reduce the average cost. The TE procedure is not able to deal with this situation, while a probabilistic procedure is.

The unit cost of heat obtained with the three procedures is not very different in this application. This is due to the particular case selected here: the zones are densely inhabited and their thermal requests are quite similar. These facts make the objective function not particularly sensible to the number of users connected with the network, except when the number of users is dramatically reduced. The area has been selected with the purpose of testing the procedure in difficult applications. In the case of different applications, in particular when the district heating in areas with rather sparse users or smaller municipalities is considered, the optimization can have a different result.

The two SA procedures provide similar results in terms of both average unit cost of heat and number of connected users. Generally speaking, the first SA method (SA1) requires less computational time and is more reliable, since the optimal configuration is found more than once with the number of performed attempts (1000, in this case). For these reasons, the application of procedure SA1 is recommended.

Nomenclature

a_i	general polynomial coefficient
A	incidence matrix
A_z	area of a zone [m^2]
b_i	general coefficient of interpolation function
c	average unit cost of heat [€/kWh]
C_C	annual capital cost [€/year]
c_F	unit cost of heat provided to the network [€/kWh]
C_{OP}	annual operating cost [€/year]
c_P	unit cost of electricity [€/kWh]
C_{net}	annual investment cost of the network [€/year]
C_{tot}	total annual cost of the network [€/year]
d	discount rate
DC	average heat unit cost difference [€/kWh]
D_{int}	the internal diameter of pipes [mm]
DTD	summation of daily temperature difference [°C·day/year]
G	water mass flow rate [kg/s]
E	energy state [J]
$\mathbf{G_x}$	vector of the general extensive property
$\mathbf{G_{xd}}$	vector accounting for the dissipations
hh	heating hours per day [h/day]
i	inflation rate
K	constant term in probability function [€/kWh]
L	pipe length [m]
L_p	annual electricity consumption [kWh/year]
p	probability of a network configuration
PC	purchase cost [€]
pl	life of the network [years]
P_0	operational cost at first year [€]
Q_F	annual thermal load [kWh/year]
Q_u	annual thermal request [kWh/year]
Q_z	annual thermal load of a zone [kWh/year]
r	volumetric specific losses per unit temperature difference [W/m^3K]
T	parameter at denominator in the probability function
U	number of buildings in a zone
v_{max}	design velocity in pipe [m/s]
V_z	total volume of buildings in the zone [m^3]
X	general size variable
Z	vector with cost rate of components [€/s]
Z_{tot}	total capital cost [€]
α	exponent in the cost function
Δp	total pressure losses [Pa]

Φ	thermal flow [kW]
γ	number of possible users in the area
η_p	average pump efficiency
λ	counter
ν	water specific volume [m^3/kg]
Π	cost rate [€/s]
Π	cost vector [€/s]
ρ	density [kg/m^3]
Ψ	exergy flow [kW]

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